Dynamics of optically generated vortices in a one–component ultracold fermionic gases

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We show that the phase imprinting method is capable of generating vortices in a one–component gas of neutral fermionic atoms at zero and finite temperatures. We find qualitative differences in dynamics of vortices in comparison with the case of Bose–Einstein condensate. The results of the imprinting strongly depend on the geometry of the trap, e.g., in asymmetric traps no single vortex state exists. These observations could be considered as a signature of Cooper–pair based superfluidity in a Fermi gas.

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One of the spectacular properties of a macroscopic bosonic–type quantum system is a superfluidity, originally observed in a liquid helium II. Although superfluidity is a complex phenomenon, it is certainly intimately related to the existence of quantized vortices. The recent experimental realization of Bose–Einstein condensation in confined alkalai–metal gases [1] allowed to study this connection in details. Quantized vortices (in a form of small arrays as well as completely ordered Abrikosov lattices) have been already observed in many laboratories [2]. Interferometric detection method showed directly 2π–phase winding associated with the presence of a vortex in the condensate [3]. Of course, this is a manifestation of the macroscopic wave function.

However, the degenerate Fermi gas has no macroscopic wave function. It is rather described (at zero temperature) in terms of the many–body wave function, built of single–particle orbitals, whose evolution is governed by the Schrödinger equation. Hence, the appearance of quantized vortices in a fermionic system demonstrates the BCS–type transition from normal to superfluid phase. Recently several groups have undertaken the effort to achieve quantum degeneracy in a dilute Fermi gas [4, 5, 6]. In JILA experiment [4, 7] ^{40}K atoms were trapped in two hyperfine states and cooled evaporatively by collisions between atoms in a different spin states whereas in Refs. [5, 6] a mixture of bosonic (^{7}Li) and fermionic (^{6}Li) atoms was used to perform the sympathetic cooling of fermions.

In this Letter we investigate the dynamics of optically generated vortices in a Fermi gas in a normal phase and compare it with the properties of vortices in the Bose–Einstein condensate. A good approximation to the spin–polarized Fermi gas at low temperatures is that of non–interacting particles (the s–wave scattering is absent for spin–polarized fermions). We then assume that at zero temperature the system is described by the Slater determinant with the lowest orbitals being occupied. The details of the technique of phase imprinting are thoroughly discussed in Refs. [8, 9] for bosons and repeated in Refs. [10, 11] for fermions. It is worth stressing that this method was already experimentally realized for the generation of solitons in Bose–Einstein condensate [12, 13].

We start our considerations with a two–dimensional symmetric trap and imprint the phase at the center of the trap. Each atom acquires a phase given by $k\phi$, where k is an integer number and ϕ is the azimuthal angle around the origin (each wave function is multiplied by the factor $\exp(ik\phi)$). The following free evolution of the single–particle orbital can be calculated by using the propagator technique

$$\varphi_j(x,y,t) = \int\int K(x,y,x',y',t)\,\varphi_j(x',y',0)\,dx'dy'\,, \tag{1}$$

where $K(x,y,x',y',t)$ is the propagator function for two–dimensional symmetric harmonic oscillator [14]

$$K(x,y,x',y',t) = \frac{M\omega}{2\pi\hbar i \sin\omega t}\exp\left\{\frac{iM\omega}{2\hbar\sin\omega t}\left[(x^2+y^2+x'^2+y'^2)\cos\omega t - 2\,xx' - 2\,yy'\right]\right\}.$$

In this case it is convenient to represent the single–particle orbitals in polar coordinates (hereafter, $\sqrt{\hbar/(M\omega)}$ and $1/\omega$ are used as units of length and time, respectively): $\varphi_{nm}(r,\phi) = A_{nm}\,r^{|m|}\,L_n^{|m|}(r^2)\,e^{-r^2/2}\,e^{im\phi}$ and $A_{nm} = (n!/[\pi(n+|m|)!])^{1/2}$ and the integration according to the prescription (1) can be performed first over the azimuthal angle then over the radial distance.

The first stage requires the use of the formula [15]

$$\int_0^{2\pi} e^{-iz\cos\Theta}\, e^{il\Theta}\, d\Theta = (-i)^l\, 2\pi\, J_l(z)\ ,\qquad (2)$$

where $J_l(z)$ is the Bessel function of integer order and z any complex number. In our case z is real and equals $rr'/\sin t$. In the second stage one has to calculate

$$\int_0^\infty r'^{(1+|m|)}\, e^{-a^2 r'^2} J_{m+k}(b\, r')\, L_n^{|m|}\, (r'^2)\, dr'\ ,\qquad (3)$$

where $a^2 = 1/2 - i\cos t/(2\sin t)$ and $b = r/\sin t$. Since the generalized Laguerre function $L_n^{|m|}$ is a polynomial, the expression (3) can be expanded and integrated term by term with the help of appropriate formula for integrals of Bessel functions [15]

$$\int_0^\infty r'^{(\mu-1)} e^{-a^2 r'^2} J_\nu(b\, r')\, dr' = \frac{\Gamma(\frac12\nu+\frac12\mu)(\frac12\frac{b}{a})^\nu}{2a^\mu\Gamma(\nu+1)}$$
$$\times M\left(\frac12\nu+\frac12\mu, \nu+1, -\frac{b^2}{4a^2}\right)\qquad (4)$$

valid for $\mathrm{Re}(a^2) > 0$ and $\mathrm{Re}(\mu+\nu) > 0$. $M(\alpha,\gamma,z)$ is a confluent hypergeometric function.

The result is the product of b^{m+k} and the combination of the confluent hypergeometric functions; each of the argument equal to $-b^2/(4a^2)$

$$\varphi(r,\phi,t) = -\frac{A_{nm}(-i)^{m+k-1}}{\sin t}\, e^{i\frac{\cos t}{2\sin t}r^2} e^{i(m+k)\phi}$$
$$\times \frac{b^{m+k}}{\Gamma(1+m+k)\, 2^{1+m+k}}$$
$$\times \sum_{l=0}^n \frac{(-1)^l}{l!}\binom{n+|m|}{n-l}\frac{\Gamma(1+l+\frac{m+k+|m|}{2})}{a^{2+2l+m+k+|m|}}$$
$$\times M\left(1+l+\frac{m+k+|m|}{2}, 1+m+k, -\frac{b^2}{4a^2}\right)\ .$$

Hence, the value of the wave function (single–particle orbital) at the origin is fully determined by the factor b^{m+k} and equals zero provided the phase imprinting is strong enough (k is larger than the maximum absolute value of magnetic number m of occupied orbitals). In such a case the diagonal part of the one–particle density matrix is zero at the origin all the time. On the other hand, the part of the phase of particular orbital dependent on the azimuthal angle is given by $(m+k)\phi$ and the circulation along the circle centered at the vortex line (point) is [10]

$$\Gamma_C(r,t) = \frac{h}{M}\frac{\sum_j |\varphi_j(r,t)|^2(m_j+k)}{\sum_j |\varphi_j(r,t)|^2}\ .$$

In Fig. 1 we plotted the circulation for various paths (circles centered at the vortex point) at different times

for a system of $N = 6$ fermions after writing the phase 3ϕ. It is clear that the strong enough phase imprinting leads to the excitation of the Fermi gas with non–zero (but not quantized) circulation around the point where the density vanishes. We call this state the vortex.
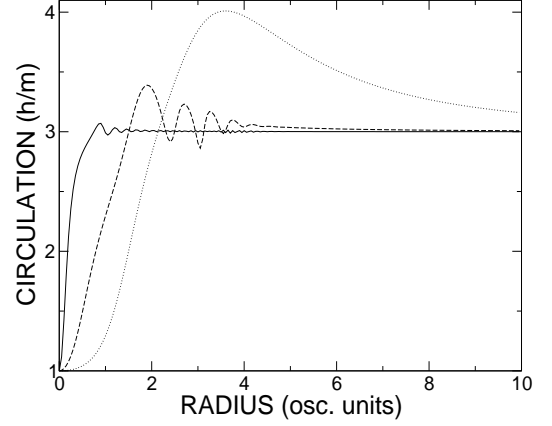

FIG. 1: Circulation along the circle as a function of its radius for various moments: 0.05 (solid line), 0.26 (dashed line), and 1.57 (dotted line) in units of $1/\omega$. The system and phase imprinting parameters are as follows: $N = 6$ and $k = 3$.

For weaker phase imprinting (e.g., for $k = 2$ and $N = 6$) the response of the system becomes more complex. The density just does not go to zero at the center. In some cases the density at the origin oscillates between zero and local maximum and the circulation goes to zero while the radius of the circle decreases (vortex–antivortex oscillations). In others, the dip in the density is strongly diminished, however the vortex reappears at multiple integers of the trap period.

It turns out that we can follow analytically the evolution of the single–particle orbitals also in the case when the absorption plate (and the laser beam) is shifted off the center of the trap. For that, the single–particle orbitals are multiplied by the factor

$$\left[\frac{(x-x_0)+i(y-y_0)}{\sqrt{(x-x_0)^2+(y-y_0)^2}}\right]^k\ ,$$

where (x_0, y_0) are coordinates of the laser beam with respect to the center of the trap. This time it is better to represent the atomic wave functions in Cartesian coordinates: $\varphi_{n_1 n_2}(x,y) = A_{n_1} A_{n_2} H_{n_1}(x) H_{n_2}(y) e^{-(x^2+y^2)/2}$, where $A_n = (1/(\pi^{1/2} 2^n n!))^{1/2}$ and $H_n(x)$ is the n–th Hermite polynomial.

To find the evolution of single–particle orbitals, governed by Eq. (1), it is convenient to shift the origin of Cartesian coordinates (x', y') by the vector (x_0, y_0) and then introduce the polar coordinates. The integration over the azimuthal angle requires the use of generating function technique and the following extension of formula

(2)

$$\int_0^{2\pi} e^{i(z\cos\phi + w\sin\phi)} e^{ik\phi}\, d\phi = \frac{2\pi i^k (z+iw)^k}{(\sqrt{z^2+w^2})^k} J_k(\sqrt{z^2+w^2})$$

with z and w being complex numbers. The result $(I_{n_1 n_2}(r'))$ for any state (n_1, n_2) is obtained by expanding the left–hand side of the equation

$$e^{-(s_1^2+s_2^2)} e^{2(s_1 x_0 + s_2 y_0)} \frac{2\pi i^k (z+iw)^k}{(\sqrt{z^2+w^2})^k} J_k(r'\sqrt{z^2+w^2}) =$$

$$\sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \frac{1}{n_1!}\frac{1}{n_2!} I_{n_1 n_2}(r')\, s_1^{n_1} s_2^{n_2} , \qquad (5)$$

where

$$z = \frac{1}{\sin t}(x_0 e^{it} - x) - i2s_1$$

$$w = \frac{1}{\sin t}(y_0 e^{it} - y) - i2s_2$$

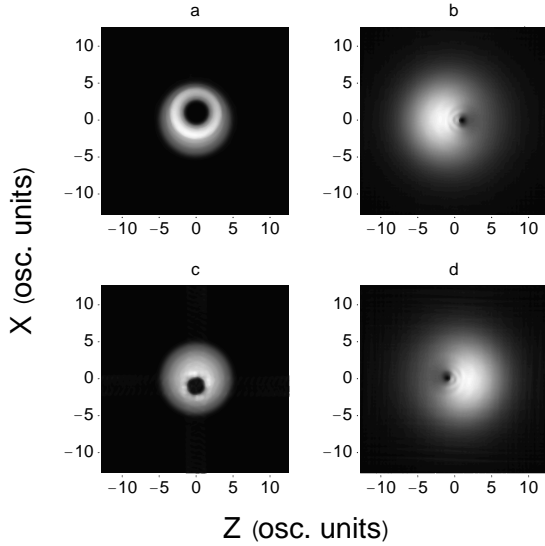in a Taylor series with respect to (s_1, s_2) around $(0,0)$ point.

FIG. 2: Evolution of the density distribution of $N = 105$ atoms in two–dimensional symmetric harmonic trap after imprinting a phase of (14ϕ) 1.5 osc. units off the center. The successive frames correspond approximately to the moments: (a) 0, (b) 1/4, (c) 1/2, and (d) 3/4 of trap period. The vortex rotates around the center of the trap changing periodically the size of its core.

Since the only dependence on the distance r' appears through the J Bessel function it is convenient to perform the integration over the radial distance first. Again, by using the formula (4) one gets (up to time–dependent constant)

$$e^{-(s_1^2+s_2^2)} e^{2(s_1 x_0 + s_2 y_0)} (z+iw)^k \qquad (6)$$

for the left–hand side of (5). The expression (6) implies that, assuming the phase imprinting is strong enough ($k > \max(n_1, n_2)$), each atomic wave function includes the factor $(z+iw)$ in an integer power higher than or equals 1, taken at the point $(s_1 = 0, s_2 = 0)$. It is easy to check that

$$(z+iw)_{s_1=0, s_2=0} = (x_0 \cos t - y_0 \sin t - x) + $$
$$i(x_0 \sin t + y_0 \cos t - y) . \qquad (7)$$

It means that for $k > \max(n_1, n_2)$ all orbitals have zero at the point $(z+iw)_{s_1=0, s_2=0} = 0$. So, this zero is also present in the diagonal part of the one–particle density matrix. Moreover, it is clear from (7) that the zero rotates around the center of the trap at the radius equal to $\sqrt{x_0^2 + y_0^2}$ with constant frequency which is the trap frequency. For $n_1 = n_2 = 0$ the time–dependent wave function is given by

$$\varphi(x, y, t) = -\frac{\Gamma(1+\frac{k}{2})}{\Gamma(1+k)} \frac{A_0 A_0 i^{k-1}}{2^{1+k} a^{2+k} \sin t} e^{i\frac{\cos t}{2\sin t}(r^2 + r_0^2) - \frac{1}{2}r_0^2}$$

$$\times e^{i\frac{1}{\sin t}(xx_0 + yy_0)} (z_1 + iw_1)^k M\left(1+\frac{k}{2}, 1+k, -\frac{b^2}{4a^2}\right) ,$$

where $z_1 = (x_0 \exp(it) - x)/\sin t$, $w_1 = (y_0 \exp(it) - y)/\sin t$, and $b^2 = z_1^2 + w_1^2$. Evolution (obtained numerically by solving the many–body Schrödinger equation [10, 11]) of the density distribution for $N = 105$ atoms is shown in Fig. 2.
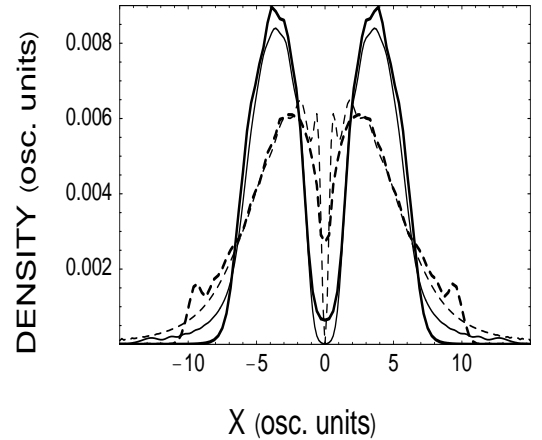

FIG. 3: Density distribution (cuts along 'x' axis) of $N = 105$ (average number) atoms at temperatures zero and $0.1 T_F$ after writing the phase 14ϕ. The curves correspond to moments 0.75 (thick and thin solid lines for $0.1 T_F$ and zero temperature, respectively) and 1.5 (thick and thin dashed lines for $0.1 T_F$ and zero temperature, respectively) in units of $1/\omega$ and are the results of averaging procedure over 1000 configurations. The thick dashed curve shows the moment of the lowest contrast (which is about 50%).

We consider now the generation and dynamics of vortices at finite temperatures. To this end, we employ the

Monte Carlo algorithm and generate a number of many–body configurations [11] according to the Fermi–Dirac statistics. At finite temperatures the single–particle states with the energy above the Fermi level are populated. It may happen that for a given phase imprint k, even strong enough to generate a vortex at zero temperature, there are configurations with such single–orbitals that $m + k = 0$. For these configurations we will see again complex structures like vortex–antivortex oscillations in the center of the trap or circulating around it, depending on the way the phase imprinting was done. Averaging procedure, according to the grand canonical ensemble rules just shows the vortex with the lower contrast (see Fig. 3). However, one might argue that in a particular experimental realization only one configuration is involved and hence vortices, reappearing vortices or vortex-antivortex oscillations should be observed. In any case, non of these structures is dissipatively circulating out of the system as predicted for the vortices in the Bose–Einstein condensate at finite temperatures [16].
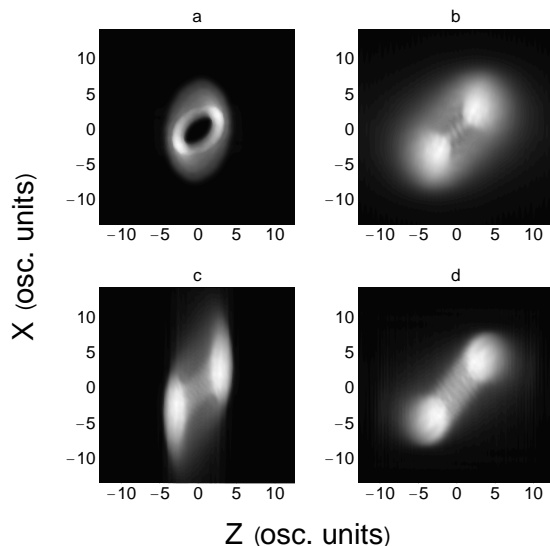

FIG. 4: Evolution of the density distribution of $N = 105$ atoms in two–dimensional asymmetric harmonic trap (30Hz × 50Hz) after imprinting a phase of $(14\,\phi)$ at the center. The successive frames correspond approximately to the moments: (a) 0, (b) 1/4, (c) 1/2, and (d) 3/4 of trap period. The vortex reappears at integer multiples of trap period.

Finally, we have done some numerical calculations for asymmetric traps at zero temperature. The results are presented in Fig. 4. Again, striking difference in comparison with the Bose–Einstein condensate case is found. No vortex is generated, presumably because there is no common zero for orbitals after phase imprinting in the case of asymmetric trap.

In conclusion, we have shown that the dynamics of vortices generated in a cold gas of neutral fermionic atoms significantly differs from the case when they are generated in the Bose–Einstein condensate. At zero temperature and symmetric harmonic trap, the vortex rotates around the center of the trap. However, for fermions it periodically changes the size of its core. For bosons, the size of the vortex core remains at the order of the healing length. Further differences appear at finite temperatures. Contrary to dissipative escape of vortex from the trap predicted for the Bose–Einstein condensate [16], we find the stable rotational motion of the fermionic vortex. Finally, the case of asymmetric trap shows new density patterns present in a Fermi gas and not observed for bosons. All these findings could be used as a test of BCS–type phase transition in a Fermi gas.

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